DETERMINATION OF RANDOM MOVEMENT CHARACTERISTIC PARAMETERS OF MECHANICAL MODELS WITH DISTRIBUTED MASS (ISOTHERMAL MODE)

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Abstract. *This paper intends to post a cinematic element Bernoulli – Euler plane moving with constant section, to obtain mathematical model displacements, which are random variables with known statistical characteristics (mathematical expectation, dispersion, correlation function) and to determine the statistical characteristics of the dynamic response.*

Keywords: *variable, random, pattern, dispersing, function, correlation*

1. INTRODUCTION

It is considered a cinematic bar element Bernoulli - Euler, moving flat, constant section (fig. 1).

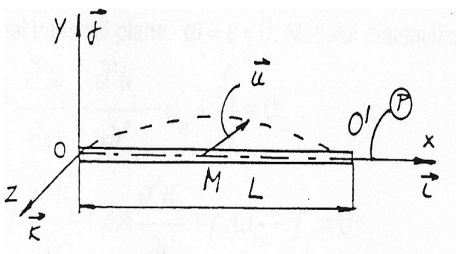

Figure 1. The cinematic bar element Bernoulli-Euler, moving flat, constant section

We use the following notations:

- $\bar{i};\bar{j};\bar{k}$ - unit vectors associated with its landmark (P);
- $\overline{a_0} = a_{01}(t)\bar{i} + a_{02}(t)\bar{j}$ - an extremity acceleration against an external reference inertial coordinate planes one of the landmark exterior (E), coinciding with the plan at any time of the landmark xOy own Cartesian (P);
- $\overline{v_0}^E = v_{01}(t)\bar{i} + v_{02}(t)\bar{j}$ - speed extremity a Cartesian coordinate system to the outside (E);
- ${}^E\bar{\omega}^P = \omega(t)\bar{k}$ - angular velocity of cinematic element in its rigid body motion;
- ${}^E\bar{\varepsilon}^P = \varepsilon(t)\bar{k} = \omega(t)\bar{k}$ - angular acceleration kinematic element, the rigid body motion;
- $\bar{f} = f_1(x,t)\bar{i} + f_2(x,t)\bar{j}$ - the external force per unit length (Nm^{-1});
- El – flexural rigidity;
- $c^2 = \dfrac{E}{\rho}$;
- E- Young's modulus;
- ρ - specific mass volumic;
- $\bar{u}(x,t) = u_1(x,t)\bar{i} + u_2(x,t)\bar{j}$ - linear-elastic displacement of a point M of the geometrical axis of the bar.

2. DEVELOPMENT SUBJECT

It neglects the influence of axial forces N and consider a mathematical model decoupled sufficiently precise current technical applications, obtained by neglecting terms intercooled.

Mathematical model is obtained in displacement:

$$c^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} + \omega^2 u_1 + v_{02}\omega - a_{01} + \omega^2 x + \frac{f_1}{\rho} = 0; \qquad (1)$$

$$El\frac{\partial^4 u}{\partial x^4} + \rho A\frac{\partial^2 u_2}{\partial t^2} + \omega^2 A\rho u_2 + \rho A a_{02} + \rho A\omega v_{01} + \rho A\varepsilon x - f_2 = 0; \qquad (2)$$

The system (1) - (2) are considered random variables with known statistical characteristics (mathematical expectation, dispersion, correlation function, etc.).

We propose to determine the statistical characteristics of the dynamic response of determining a priori adopting STI strategy.

The homogeneous initial conditions are:

$$u_1(x,0) = \frac{\partial u_1}{\partial t}(x,0) = u_2(x,0) = \frac{\partial u_2}{\partial t}(x,0) = 0 \qquad (3)$$

and limit conditions are:

$$u_1(0,t) = u_1(L,t) = 0; u_2(0,t) = \frac{\partial^2 u_2}{\partial x^2}(0,t) = u_2(L,t) = \frac{\partial^2 u_2}{\partial x^2}(L,t) = 0 \qquad (4)$$

expressing the double articulation of the bar at the ends A and A' of rigid kinematic elements.

In order not to unnecessarily complicate the process description, it is considered the particular case of a plane translation: $\omega = \varepsilon = 0$.

The mathematical model (1) - (2) becomes:

$$c^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u_1}{\partial t^2} - a_{01} + \frac{f_1}{\rho} = 0 \qquad (5)$$

$$EI\frac{\partial^4 u_2}{\partial x^4}+\rho A\frac{\partial^2 u_2}{\partial t^2}+\rho A a_{02}-f_2=0 \qquad (6)$$

Applying in (5) and (6) the Laplace transform with respect to time and Fourier finite sine compared to x, we obtain an algebraic system, after solving and reverse successive integral transformations, mathematical model provides the solution:

$$u_1(x,t)=\frac{2}{L}\sum_{n=1}^{\infty}\left\{\frac{c\alpha_n}{\rho}\times\int_0^1(n,\tau)\sin c\alpha_{n(}(t-\tau)d\tau-\frac{Lc\alpha_n\left[1+(-1)^{n+1}\right]}{n\pi}\times\right.$$
$$\left.\times\int_0^t a_{01}\sin c\alpha_n(t-\tau)d\tau\right\}\sin(\alpha_n x) \qquad (7)$$

$$u_2(x,t)=\frac{2}{\rho AL}\sum_{n=1}^{\infty}\left\{\sqrt{\frac{EI}{\rho A}}\alpha_n^2\int_0^t f_2^n(n,\tau)\sin\sqrt{\frac{EI}{\rho}}\alpha_n^2(t-\tau)d\tau-\right.$$
$$\left.-\sqrt{\frac{EI}{\rho A}}\alpha_n^2\rho A\frac{L}{n\pi}\left[1+(-1)^{n+1}\right]\times\int_0^t a_{02}(\tau)\sin\sqrt{\frac{EI}{\rho}}\alpha_n^2(t-\tau)-d\tau\right\}\sin(\alpha_n x) \qquad (8)$$

We use the following notations:

- $n\in N;\ a_n=\frac{np}{L};$
- $f_{1,2}^{*}(n,\tau)=\int_0^t f_{1,2}(x,t)\sin(\alpha_n x)dx.$

It is obvious that m_{f_1}' and m_{f2}' data are obtained from m_{f1} and m_{f2} with relationships definition of finite Fourier sine:

$$m_{f'_{1,2}}=\int_0^L m_{f_{1,2}}\sin(\alpha_n x)\ \mathrm{x}\ dx. \qquad (9)$$

If $m_{f_{1,2}}$ are time functions, is obtained:

$$m_{f'_{1,2}}=m_{f_{1,2}}\frac{1-(-1)^{n+1}}{\alpha_n} \qquad (10)$$

Using the definition of mathematical expectancy are obtained mathematical hopes of u_1 and u_2 (11), (12):

$$m_{u_1}=\frac{2}{L}\sum_{n=1}^{\infty}\{\frac{c\alpha_n}{\rho}\mathrm{x}\int_0^t m'_{f_1}(\tau)\sin c\alpha_n(t-\tau)d\tau-$$
$$\frac{Lc\alpha_n[1+(-1)^{n+1}]}{n\pi}\mathrm{x}\int_0^t m_{a_{01}}(\tau)\sin c\alpha_n(t-\tau)d\tau\}\sin(\alpha_n x) \qquad (11)$$

$$m_{u_2}=\frac{2}{\rho AL}\sum_{n=1}^{\infty}\{\sqrt{\frac{EI}{\rho A}}\alpha_n^2\int_0^t m_{f'_2}(\tau)\sin\sqrt{\frac{EI}{\rho}}\alpha_n^2(t-\tau)d\tau-$$
$$\sqrt{\frac{EI}{\rho A}}\alpha_n^2\rho A\frac{L}{n\pi}[1+(-1)^{n+1}]\mathrm{x}\int_0^t m_{a_{02}}(\tau)\sin\sqrt{\frac{EI}{\rho}}\alpha_n^2(t-\tau)d\tau\}\sin(\alpha_n x) \qquad (12)$$

It is known correlation functions

$K_{f'_1}(\tau,\tau'),\ K_{a_{01},f'_1}(\tau,\tau'),\ K_{a_{02}}(\tau,\tau'),\ K_{f'_2}(\tau,\tau'),\ K_{a_{02},f'_2}(\tau,\tau').$

We use the following notations (13):

$S_1(x,\tau,t)=\frac{2}{L}\sum_{n=1}^{\infty}\frac{c\alpha_n}{\rho}\sin c\alpha_n(t-\tau)\sin(\alpha_n x);$

$S_2(x,\tau,t)=-\frac{2}{L}\sum_{n=1}^{\infty}\frac{Lc\alpha_n[1+(-1)^{n+1}]}{n\pi}\sin c\alpha_n(t-\tau)\sin(\alpha_n x);$

$S_3(x,\tau,t)=\frac{2}{\rho AL}\sum_{n=1}^{\infty}\sqrt{\frac{EI}{\rho A}}\alpha_n^2\sin\sqrt{\frac{EI}{\rho}}\alpha_n^2(t-\tau)\sin(\alpha_n x);$

$S_4(x,\tau,t)=-\frac{2}{\rho AL}\sum_{n=1}^{\infty}\sqrt{\frac{EI}{\rho A}}\alpha_n^2\rho A\frac{L}{n\pi}[1+(-1)^{n+1}]\sin\sqrt{\frac{EI}{\rho}}\alpha_n^2(\sin(\alpha_n x)$ (13)

Using the definitions of the functions of correlation (or auto correlation respectively) was obtained following formulas (14), (15):

$K_{u_1}(t,t')=\int_0^t\int_0^{t'}\{[m_{f'_1}(\tau)\,m_{f'_1}(\tau')+K_{f'_1}(\tau,\tau')]\mathrm{x}S_1(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_1(x,\tau',t')+[m_{a_{01}}(\tau)\,m_{f'_1}(\tau')+K_{a_{01},f'_1}(\tau,\tau')]\ S_1(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_1(x,\tau',t')+[m_{f'_1}(\tau)\,m_{a_{01}}(\tau')+K_{f'_1,a_{01}}(\tau,\tau')]\ S_1(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_2(x,\tau',t')+[m_{a_{01}}(\tau)\,m_{a_{01}}(\tau')+K_{a_{01}}(\tau,\tau')]\ S_2(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_2(x,\tau',t')-[m_{f'_1}(\tau)\ S_1(x,\tau,t)+m_{a_{01}}(\tau)\ S_2(x,\tau,t)]\mathrm{x}$
$\mathrm{x}[m_{f'_1}(\tau)\ S_1(x,\tau,t')+m_{a_{01}}(\tau)\ S_2(x,\tau,t')]-$
$[m_{f'_1}(\tau)\ S_1(x,\tau,t')+m_{a_{01}}(\tau)\ S_2(x,\tau,t')]\mathrm{x}$
$[m_{f'_1}(\tau)\ S_1(x,\tau,t)+m_{a_{01}}(\tau)\ S_2(x,\tau,t)]+$
$[m_{f'_1}(\tau)\ S_1(x,\tau,t)+m_{a_{01}}(\tau)\ S_2(x,\tau,t)]\mathrm{x}$
$[m_{f'_1}(\tau)\ S_1(x,\tau,t')+m_{a_{01}}(\tau)\ S_2(x,\tau,t')]\}\,dt\,dt'$

(14)

$K_{u_2}(t,t')=\int_0^t\int_0^{t'}\{[m_{f'_2}(\tau)\,m_{f'_2}(\tau')+K_{f'_2}(\tau,\tau')]\mathrm{x}S_3(x,\tau,t)\mathrm{x}$
$S_3(x,\tau',t')+[m_{a_{02}}(\tau)\,m_{f'_2}(\tau')+K_{a_{02},f'_2}(\tau,\tau')]\ S_4(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_3(x,\tau',t')+[m_{f'_2}(\tau)\,m_{a_{02}}(\tau')+K_{f'_2,a_{02}}(\tau,\tau')]\ S_3(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_4(x,\tau',t')+[m_{a_{02}}(\tau)\,m_{a_{02}}(\tau')+K_{a_{02}}(\tau,\tau')]\ S_4(x,\tau,t)\mathrm{x}$
$\mathrm{x}S_4(x,\tau',t')-[m_{f'_2}(\tau)\ S_3(x,\tau,t)+m_{a_{02}}(\tau)\ S_4(x,\tau,t)]\mathrm{x}$
$[m_{f'_2}(\tau)\ S_3(x,\tau,t')+m_{a_{02}}(\tau)\ S_4(x,\tau,t')]-$
$[m_{f'_2}(\tau)\ S_3(x,\tau,t')+m_{a_{02}}(\tau)\ S_4(x,\tau,t')]\mathrm{x}$

$[m_{f_2'}(\tau)\, S_3(x,\tau,t) + m_{a_{oz}}(\tau)\, S_4(x,\tau,t)] +$

$[m_{f_2'}(\tau)\, S_3(x,\tau,t) + m_{a_{oz}}(\tau)\, S_4(x,\tau,t)]\mathrm{x}$

$[m_{f_2'}(\tau)\, S_3(x,\tau,t') + m_{a_{oz}}(\tau)\, S_4(x,\tau,t')]\}\, dt\, dt'$

(15)

Dispersions will be:

$D_{u_1}(x,t) = K_{u_1}(t,t),\ D_{u_2}(x,t) = K_{u_2}(t,t)$ (16)

Manner is the same as for other types of instant rototranslation cinematic element, including spatial movement.

3. CONCLUSION

As seen in the example treated, the actual calculation of the statistical characteristics of the dynamic response – when the mathematical model is a system of partial differential equations – is difficult, especially when it is considered decoupling of the equations of motion of the model and the condition rigid solid is complex.

4. REFERENCES

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